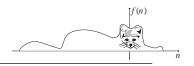
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## Exercise set 6 Tue Nov 2.2010 14.30-16.00 MaD-355

## Topological vector spaces

- **6.1.** Prove that the weak topology  $\sigma(E, F)$  is Hausdorff if and only if the duality separates E:n.
- **6.2.** Provet that if E is a locally convex Hausdorff-space, then  $\sigma(E, E^*)$  is Hausdorff-topology.
- **6.3.** We call a **locally convex** topology  $\tau$  of E compatible with the duality (E, F), if

$$E_{\tau}^* = F$$
.

. For example if E is locally convex Hausdorff-space, the weak toplogy  $\sigma(E, E^*)$  is compatible with  $(E, E^*)$ . Also the original topology of E of course is compatible. Is  $\sigma(E, E^*)$  the finest — or maybe the roughest — compatible with  $(E, E^*)$  topology?

- **6.4.** Assume that (E, F) a separ dual pair.
- a) Prove that a balanced, convex set A has the same closure in all topologies compatible with the dual pair (E, F).
- b) In fact, it is redundant to assume that A is balanced. (Hint : Banachin sep thm..)
  - c) Does every topology compatible with  $(E, E^*)$  have the same barrels? Find a condition which is ewuivalent to the existence of a separ. dual pair (E, F).
- **6.5.** Assume that E topological vector space and  $E^*$  its topological dual. Prove that if the dual pair  $(E, E^*)$  separates E, then E is a Hausdorff–space.
- **6.6.** Let E and F ne vector spaces, dim  $E < \infty$ .. Find a condition equivalent to the existence of a separ duality (E, F).
- **6.7.** Assume that E has the topology  $\sigma(E, E')$  (algebraic dual!). Prove that if  $A \subset E$  is bounded, then a) there exist a finite dimensional subspace  $G \subset E$  such, that  $A \subset G$ 
  - b) E: s every vector subspace is closed
  - c) E: s every vector subspace ha a topological supplement.
- **6.8.** Assume that E is an infinitely dimensional locally convex Hausdorff-avaruus. Prove that  $E^*_{\sigma}$  is not normable.
- **6.9.** Assume that E is an infinitely dimensional normed space. Prove that the zero vector of the dual  $: 0 \in E^*$  belongs to the closure of  $\{x' \mid ||x'|| = 1\}$  in the topology  $\sigma(E^*, E)$ .