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Exercise set 6 **Topological vector spaces**
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6.1. Prove that the weak topology $\sigma(E, F)$ is Hausdorff if and only if the duality separates E:n.

6.2. Provet that if E is a locally convex Hausdorff-space, then $\sigma(E, E^*)$ is Hausdorff-topology.

6.3. We call a **locally convex** topology τ of E *compatible with the duality* (E, F), if

$$E^*_\tau = F.$$

. For example if E is locally convex Hausdorff-space, the weak toplogy $\sigma(E, E^*)$ is compatible with (E, E^*). Also the original topology of E of course is compatible. Is $\sigma(E, E^*)$ the finest — or maybe the roughest — compatible with (E, E^*) topology?

6.4. Assume that (E, F) a separ dual pair.

a)Prove that a balanced, convex set A has the same closure in all topologies compatible with the dual pair (E, F).

b) In fact, it is redundant to assume that A is balanced. (Hint : Banachin sep thm..)

c) Does every topology compatible with (E, E^*) have the same barrels ?

Find a condition which is ewuivalent to the existence of a separ. dual pair (E, F).

6.5. Assume that E topological vector space and E^* its topological dual. Prove that if the dual pair (E, E^*) separates E, then E is a Hausdorff–space.

6.6. Let E and F ne vector spaces, $\dim E < \infty$.. Find a condition equivalent to the existence of a separ duality (E, F).

6.7. Assume that E has the topology $\sigma(E, E')$ (algebraic dual!). Prove that if $A \subset E$ is bounded, then a) there exist a finite dimensional subspace $G \subset E$ such, that $A \subset G$

b) E: s every vector subspace is closed

c) E: s every vector subspace ha a topological supplement.

6.8. Assume that E ia an infintely dimensional locally convex Hausdorff-avaruus.Prove that E^*_σ is not normable.

6.9. Assume that E is an infintely dimensional normed space.Prove that the zero vector of the dual : $0 \in E^*$ belongs to the closure of $\{x' \mid \|x'\| = 1\}$ in the topology $\sigma(E^*, E)$.